SEQUENCING PROBLEM

Determine the order or sequence in which the jobs are to be processed through machines so as to minimize the total processing time. Here the total effectiveness, which may be the time or cost that is to be minimized is the function of the order of sequence. Such type of problem is known as **SEQUENCING PROBLEM**

Types of Sequencing Problems

(a)'n' jobs are to be processed on **two machines** say machine A and machine B in the order AB. This means that the job is to be processed first on machine A and then on machine B.

(b)'n' jobs are to be processed on **three machines** A,B and C in the order ABC i.e. first on machine A, second on machine B and third on machine C.

(c)'n' jobs are to be processed **on 'm' machines** in the given order

(d) Two jobs are to be processed **on 'm' machines** in the given order.

SEQUENCING OF 'N' JOBS ON THREE MACHINES

(a) Identify the smallest time element in the first column, i.e. for machine 1 let it be A_r.

(b) Identify the smallest time element in the third column, i.e. for machine 3, let it be C_s

(c) Identify the highest time element in the second column, i.e. for the center machine, say machine 2, let it be B_i.

(d)Now minimum time on machine 1 i.e. A_r must be ≥ maximum time element on machine 2, i.e. B_i OR Minimum time on third machine i.e. C_s must be ≥ maximum time element on machine 2 i.e. B_i OR

Both A_r and C_s must be $\geq B_i$

(e)If the above condition satisfies, then we have to workout the time elements for two hypothetical machines, namely machine G and machine H. The time elements for machine G, $\mathbf{G_i = A_i + B_i}$.

The time element for machine H, is $\mathbf{H_i = B_i + C_i}$

(f)Now the three-machine problem is converted into two-machine problem. We can find sequence by applying Johnson Bellman rule.

N’ Jobs and Two Machines

Johnson and Bellman algorithm for optimal sequence identify the smallest element in the given matrix.

If the smallest element falls under column 1 i.e under machine I then do that job first.

If the smallest element falls under column 2 i.e under machine 2 then do that job last.

If there is tie i.e we have smallest element of same value in both columns, then:

(i) Minimum of all the processing times is A_r which is equal to B_s i.e. Min $(A_i, B_i) = A_r = B_s$ then do the r th job first and s th job last.

N’ Jobs and Two Machines

(ii) If Min (A_i, B_i) = A_r and also A_r = A_k (say). Here tie occurs between the two jobs having same minimum element in the same column i.e. first column we can do either r th job or k th job first. There will be two solutions. When the ties occur due to element in the same column, then the problem will have alternate solution. If more number of jobs have the same minimum element in the same column, then the problem will have many alternative solutions

N' Jobs and 'M' Machines

A general sequencing problem of processing of 'n' jobs through 'm' machines M_1, M_2, M_3, ………M_{n-1} , M_n in the order M_1, M_2, M_3 …… M_{n-1}, M_n can be solved by applying the following rules.

If a_{ij} where I = 1,2,3…..n and j = 1, 2, 3……….m is the processing time of i th job on j th machine, then find Minimum a_{i1} and Min. a_{im} (i.e. minimum time element in the first machine and i

minimum time element in last Machine) and find Maximum a_{ij} of intermediate machines i.e 2 nd machine to m-1 machine. i

The problem can be solved by converting it into a two-machine problem if the

following conditions are satisfied.

N' Jobs and 'M' Machines

(a) Min $a_{i1} \geq$ Max. aij $^{\text{for all j = 1,2,3,.....m-1}}$

(b) Min $a_{im} \geq$ Max aij $^{\text{for all j = 1, 2,3 ....m-1}}$

i

At least one of the above must be satisfied. Or both may be satisfied. If satisfied, then the problem can be converted into 2- machine problem where Machine G = $a_{i1} + a_{i2} + a_{i3}$ +…………+ $a_{i\ m-1}$ and

- Machine G = $a_{i2} + a_{i3}$ +………. + a_{im}. Where i = 1,2,3,….n.
- Once the problem is a 2- machine problem, then by applying Johnson Bellman algorithm we can find optimal sequence and then workout total elapsed time as usual.

2 - JOBS ON ‘M‘ MACHINES

Graphical Method

This method is applicable to solve the problems involving 2 jobs to be processed on ‘m‘ machines in the given order of machining for each job. In this method the procedure is:

(a)Represent Job 1 on X- axis and Job 2 on Y-axis. We have to layout the jobs in the order of machining showing the processing times.

(b)The **horizontal line** on the graph shows the processing time of **Job 1** and idle time of Job 2. Similarly, a **vertical line** on the graph shows processing time of **job 2** and idle time of job 1. Any inclined line shows the processing of two jobs simultaneously.

(c)Draw horizontal and vertical lines from points on X- axis and Y- axis to construct the blocks and hatch the blocks. (Pairing of same machines).

2 - JOBS ON ‘M‘ MACHINES

(d) Our job is to find the minimum time required to finish both the jobs in the given order of

machining. Hence we have to follow inclined path, preferably a line inclined at 45 degrees

(in a square the line joining the opposite coroners will be at 45 degrees).

(e)While drawing the inclined line, care must be taken to see that it will not pass through the region indicating the machining of other job. That is the inclined line should not pass through blocks constructed in step (c).

(f)After drawing the line, the total time taken is equals to Time required for processing + idle time for the job.

The sum **of processing time + idle time** for both jobs must be same

Problem

A machine operator has to perform two operations, turning and threading, on a number of different jobs. The time required to perform these operations in minutes for each job is given. Determine the order in which the jobs should be processed in order to minimize the total time required to turn out all the jobs.

Jobs: 1 2 3 4 5 6

Time for turning (in min.) 3 12 5 2 9 11

Time for threading (in 1 9 6 3

min).8 0 1

Queuing Theory

Queuing theory, one has to understand two things in clear. They are service and customer or element. Here customer or element represents a person or machine or any other thing, which is in need of some service from servicing point. Service represents any type of attention to the customer to satisfy his need. For example,

1.Person going to hospital to get medical advice from the doctor is an element or a customer,

2.A person going to railway station or a bus station to purchase a ticket for the journey is a customer or an element,

3. A person at ticket counter of a cinema hall is an element or a customer,

Service means

1. Doctor is a service facility and medical care is a service,
2. Ticket counter is a service facility and issue of ticket is service.
3. Ticket counter is a service facility and issue of ticket is service.
4. Shop owner is a service facility and issue of items is service.
5. Bank clerk is a service facility and passing the cheque is service

QUEUING SYSTEM

One thing we have to remember is that when we speak of queue, we have to deal with two elements, i.e. Arrivals and Service facility. Entire queuing system can be completely described by:

(a) The input (Arrival pattern)

(b) The service mechanism or service pattern,

(c) The queue discipline and

(d) Customer behavior.

Customer behaviour

The length of the queue or the waiting time of a customer or the idle time of the service facility mostly depends on the behaviour of the customer. Here the behaviour refers to the impatience of a customer during the stay in the line. Customer behaviour can be classified as:

(i) **Balking**: This behaviour signifies that the customer does not like to join the queue seeing the long length of it. This behaviour may effect in loosing a customer by the organization.

(ii) Reneging: In this case the customer joins the queue and after waiting for certain time looses his patience and leaves the queue. This behaviour of the customer may also cause loss of customer to the organization.

(iii)Collusion: In this case several customers may collaborate and only one of them may stand in the queue. One customer represents a group of customer. Here the queue length may be small but service time for an individual will be more. This may break the patience of the other customers in the waiting line and situation may lead to any type of worst episode.

(iv) Jockeying: If there are number of waiting lines depending on the number of service stations, for example Petrol bunks, Cinema theaters, etc. A customer in one of the queue after seeing the other queue length, which is shorter, with a hope of getting the service, may leave the present queue and join the shorter queue.

Queue discipline or Service discipline

When the customers are standing in a queue, they are called to serve depending on the nature of the customer. The order in which they are called is known as Service discipline. There are various ways in which the customer called to serve. They are:

(i) First In First Out (FIFO) or First Come First Served (FCFS)

(ii) Last in first out (LIFO) or Last Come First Served (LCFS)

(iii) Service In Random Order (SIRO)

(iv) Service By Priority

QUEUE MODELS

Most elementary queuing models assume that the inputs / arrivals and outputs / departures follow a birth and death process. Any queuing model is characterized by situations where both arrivals and departures take place simultaneously. Depending upon the nature of inputs and service faculties, there can be a number of queuing models as shown below:

(i) **Probabilistic queuing model:** Both arrival and service rates are some unknown random variables.

(ii) **Deterministic queuing model**: Both arrival and service rates are known and fixed.

M / M / 1, Model

<u>Poisson Arrival / Poisson output / Number of channels / Infinite capacity</u>

<u>/ FIFO Model :</u> M / M / 1 / (∞ / FIFO):

<u>Formulae used</u>

1. Average number of arrivals per unit of time = λ
2. Average number of units served per unit of time = μ
3. Traffic intensity or utility ratio $\rho = \frac{\lambda}{\mu}$ the condition is (λ>μ)
4. Probability that the system is empty $= P_0 = (1-\rho)$
5. Probability that there are ‘n’ units in the system = $P_n = \rho^n . P_0$

6. Average number of units in the system

= $E(n) = \frac{\rho}{(1-\rho)}$ or $= \frac{\lambda}{(\lambda-\mu)} = L_q + \frac{\lambda}{\mu}$

7. Average number of units in the waiting line

= $E_L = \frac{\rho^2}{(1-\rho)}$

8. Average waiting length (mean time in the system) =

= $\frac{1}{\mu-\lambda} = \frac{L}{\lambda}$

9. Average length of waiting line with the condition that it is always greater than zero = $V(n) = \frac{\rho}{(1-\rho)^2} = \frac{\lambda}{(\lambda-\mu)^2} = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu-\lambda)}$

10. Average time an arrival spends in the system = $E(v) = \frac{1}{\mu(1-\rho)} = \frac{1}{(\mu-\lambda)}$

11. System is busy $= \rho$

12. Idle time = $(1-\rho)$

Little's queuing formula

For *any* queuing system in which a steady-state distribution exists, the following relations hold:

$$L = \lambda W$$

$$L_q = \lambda W_q \qquad L_s = \lambda W_s$$

$$L = \frac{\rho}{1-\rho} = \frac{\lambda}{(\mu - \lambda)}$$

We determine,

$$W = \frac{L}{\lambda} = \frac{1}{(\mu - \lambda)} \qquad W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{\rho}{(\mu - \lambda)} \qquad W_s = \frac{L_S}{\lambda} = \frac{1}{\mu}$$

$\lambda =$ average number of arrivals *entering* the system per unit time

$L =$ average number of customers present in the queuing system

$L_q =$ average number of customers waiting in line

$L_s =$ average number of customers in service

$W_s =$ average time a customer spends in the system

$W_q =$ average time a customer spends in line

$W_s =$ average time a customer spends in service

Numerical

In a departmental store one cashier is there to serve the customers. And the customers pick up their needs by themselves. The arrival rate is 9 customers for every 5 minutes and the cashier can serve 10 customers in 5 minutes. Assuming Poisson arrival rate and exponential distribution for service rate, find:

(a) Average number of customers in the system.

(b) Average number of customers in the queue or average queue length.

(c) Average time a customer spends in the system.

(d) Average time a customer waits before being served.