PROJECT SCHEDULING

&

NETWORK ANALYSIS (CPM & PERT)

What is a Project?

- A project is an interrelated set of activities that has definite starting and ending points and that result in a unique product or service
- Cuts across organizational lines they need varied skills of different profession
- Uncertainties like new technology & external environment can change the character of the project
- Personnel, materials, facilities etc. are temporarily assembled to accomplish a goal within a specified time frame and then disbanded
- Upon finish, a project releases lot of resources which were engaged in execution of the project

Examples of Project

- □Hosting a College Annual Function
- \Box Plan a Space Shuttle to Mars
- Construct a Plant to Manufacture Ball Bearings
- □Plan for Wedding
- Designing and Implement a Computer System
- Designing a ABS System
- Executing Environmental Clean-up Of a Contaminated Site
- Erect a New Lab in the Dept. of Mechanical Engineering

Definition of Project

- A project is a one shot, time limited, goal directed, major undertaking, requiring the commitment of varied skills & resources.
- It also describes project as a combination bfuman and non human resources pooled together in a temporary organization to achieve specific purpose

Project Attributes

- A project:
 - •Has a unique purpose.
 - •Is temporary.
 - •Is developed using progressive elaboration.
 - •Requires resources, often from various areas.
 - •Should have a primary customer or sponsor.
 - The project sponsor usually provides the direction and funding for the project.
 Involves uncertainty.

Project and Program Managers

Project managers work with project sponsors, project teams, and other people involved in projects to meet project goals.

Program: "A group of related projects managed in a coordinated way to obtain benefits and control not available from managing them individually."

Program managers oversee programs and often act as bosses for project managers.

Project management is "the application of knowledge, skills, tools and techniques to project activities to meet project requirements."

Project Management Tools and Techniques

Project management tools and techniques assist project managers and their teams in various aspects of project management.

Specific tools and techniques include:

Project charters, scope statements, and WBS (scope).

Gantt charts, network diagrams, critical path analyses, critical chain scheduling (time).

Cost estimates and earned value management (cost).

Network Analysis refers to a number of techniques for the planning and control of complex projects.

The two most frequently used forms of network planning are:

- Programme Evaluation and Review Technique (PERT)
- 2. Critical Path Method (CPM)

WHAT IS A NETWORK?

A network is a graphical diagram consisting of certain configuration of "Arrows" (\Box) and "Nodes" (\bigcirc) for showing the logical sequence of various tasks to be performed to achieve the project objective.



PERT / CPM Techniques

The initial step in PERT/CPM project scheduling process is the determination of all specific activities that comprise the project and their relationships.

EXAMPLE

Activity	Description	Duration (in weeks)	Immediate predecessor
A	Obtain the budget approval	2	_
В	Obtain the machine	5	А
С	Hire the operator	1	Α
D	Install the machine	1	В
E	Train the operator	6	С
F	Produce the goods	1	D,E

TERMS USED IN A NETWORK

1. <u>Activity:</u> An effort that is required to complete a part of the project. It is represented by " \square ".

2. <u>Node:</u> It represents the beginning or completion of an activity. It is represented by '□'

RULES OF NETWORK CONSTRUCTION

- 1. Each defined activity is represented by one and **only one arrow** in the network.
- 2. Before an activity can be undertaken, all activities preceding it must be completed.
- 3. The **arrows** depicting various **activities** are indicative of **logical procedure** only. The length and bearing of the arrows are of no significance.

- 4. The **arrow direction** indicates the general **progression** in time. Head events and Tail events.
- 5. When a number of activities terminate at one event, it indicates that no activity emanating from that event may start unless all activities terminating there have been completed.
- 6. Events are identified by **numbers**.
- 7. The activities are identified by the numbers of their starting and ending events or by alphabets.

8. A network should have only one initial and terminal node.



Merge Event

Burst Event

10. **Parallel activities** between two events, without intervening events, are **prohibited**. When two or more parallel activities in a project have the same head and tail events, dummy activities are needed in constructing the network.

Dummy activities do not consume time or resources. An efficient network contains a minimum number of dummy activities required to portray the correct precedence relationships.

11. Looping is not permitted in a network.

NETWORK SYMBOLS

SYMBOL

MEANING

Activity

Event

Activity A must be completed before Activity B completed

Activities A & B can occur concurrently, but both must be completed before activity C can begin

Activities A & B must be completed before activities C & D can begin, but C can begin independently of D & vice versa

SAMPLE NETWORK



DUMMY ACTIVITY



CRITICAL PATH METHOD

CPM aims at the determination of the time to complete a project and the important activities on which a manager shall focus attention.

PROCEDURE

□ Consider all the paths in a project, beginning with the start event and stopping at the end event.

☐ For each path, calculate the time of execution.

□ The path with the largest time is called the critical path and the activities along this path are called critical activities or bottleneck activities.

 You are required to prepare a network diagram for constructing a 5 floor apartmer The major activities of the project are given as follows:

Activity	Description	Immediate Predecessor
А	Selection of site	5
В	Preparation of drawings	。 」
С	Arranging the for finance	А
D	Selection of contractor	А
E	Getting approval from Govt	A
F	Laying the foundation	E
G	Start construction	D, F
Н	Advertise in newspaper	B, C
I	Allocation of tenants	G, H
8		



PERT (Project or Program Evaluation and Review Techniques)

PROJECT EVALUATION REVIEW TECHNIQUE

In the critical path method, the time estimates are assumed to be known with certainty. In certain projects like research and development, new product introductions, it is difficult to estimate the time of various activities.

Hence PERT is used in such projects with a probabilistic method using three time estimates for an activity, rather than a single estimate, as shown in Figure Optimistic time to



Optimistic time t₀:

It is the shortest time taken to complete the activity. It means that if everything goes well then there is more chance of completing the activity within this time.

Most likely time tm:

It is the normal time taken to complete an activity, if the activity were frequently repeated under the same conditions.

Pessimistic time tp:

It is the longest time that an activity would take to complete. It is the worst time estimate that an activity would take if unexpected problems are faced.

Figure 8.22: PERT Using Probabilistic Method with 3 Time Estimates

Taking all these time estimates into consideration, the expected time of an activity is arrived at.

The average or mean (ta) value of the activity duration is given by,

The variance of the activity time is calculated using the formula,

The probability of completing the project within the scheduled time (Ts) or contracted time may be obtained by using the standard normal deviate where Te is the expected time of project completion.

Probability of completing the project within the scheduled time is,

$$T_{a} = \frac{t_{0} + 4t_{m} + t_{p}}{6} \qquad(5)$$
$$\sigma_{1}^{2} = \left(\frac{t_{p} - t_{0}}{6}\right)^{2}$$

$$Z_0 = \frac{T_s - T_e}{\sqrt{\Sigma \sigma^2} \text{ in critical path}}$$

 $P(T \le T_s) = P(Z \le Z_0)$ (from normal tables)

Example Problem of PERT

An R &D project has a list of tasks to be performed whose time estimates are given in the Table 8.11, as follows.

Activity	Activity Name	T ₀	tm	tp
i j			(in days)	
1-2	А	4	б	8
1-3	В	2	3	10
1-4	С	б	8	16
2-4	D	1	2	3
3-4	Е	б	7	8
3-5	F	б	7	14
4-6	G	3	5	7
4-7	Н	4	11	12
5-7	I	2	4	6
6-7	J	2	9	10

Table 8.11: Time Estimates for R & D Project

a. Draw the project network.

b. Find the critical path.

c.Find the probability that the project is completed in 19 days. If the probability is less than 20%, find the probability of completing it in 24 days.

Time expected for each activity is calculated using the formula (5): Similarly, the expected time is calculated for all the activities. $T_{a} = \frac{t_{0} + 4tm + tp}{6}$ $= \frac{4 + 4(6) + 8}{6} = \frac{36}{6} = 6 \text{ days for activity A}$

The variance of activity time is calculated using the formula (6). Similarly, variances of all the activities are calculated.

$$\sigma_i^2 = \left(\frac{t_p - t_0}{6}\right)^2$$

$$=\left(\frac{8-4}{6}\right)^2 = 0.444$$

Table 8.12: T & s² Calculated

Activity	To	Tm	Tp	Ta	σ^2
1-2	4	6	8	б	0.444
1-3	2	3	10	4	1.777
<mark>(1-4</mark>)	6	8	16	9	2.777
2-4	1	2	3	2	0.111
3-4	6	7	8	7	0.111
3-5	6	7	14	8	1.777
4 <mark>-6</mark>	3	5	7	5	0.444
4-7	4	11	12	10	1.777
5-7	2	4	6	4	0.444
<mark>6</mark> -7	2	9	10	8	1.777

A) Construct a network diagram:



calculate the time earliest (TE) and time Latest (TL) for all the activities.

From the network diagram Figure 8.24, the critical path is identified as 1-4, 4-6, 6-7, with a project duration of 22 days.

C) The probability of completing the project within 19 days is given by, P (Z<Z0)

To find Z0,
$$Z_0 = \left(\frac{T_s - T_e}{\sqrt{\Sigma\sigma \text{ in critical path}}}\right)$$
$$= \left(\frac{19 - 22}{\sqrt{2.777 + 0.444 + 1.777}}\right) \qquad = \left(\frac{-3}{\sqrt{5}}\right) = -1.3416$$

we know, P (Z <Z Network Model o) = 0.5 - z (1.3416) (from normal tables, z (1.3416) = 0.4099) = 0.5 - 0.4099= 0.0901= 9.01%

Thus, the probability of completing the R &D project in 19 days is 9.01%.

Since the probability of completing the project in 19 days is less than 20% As in question, we find the probability of completing it in 24 days.

$$Z_{0} = \frac{T_{s} - T_{e}}{\sqrt{\Sigma\sigma \text{ in critical path}}}$$

= $\left(\frac{24 - 22}{\sqrt{5}}\right) = \left(\frac{2}{\sqrt{5}}\right) = 0.8944 \text{ days}$
P $(Z \le Z_{0}) = 0.5 - Y \ (0.8944)$ (from normal tables, Y $(0.8944) = 0.3133$)
= $0.5 + 0.3133$
= 0.8133

= 81.33%

COST ANALYSIS

The two important components of any activity are the cost and time. Cost is directly proportional to time and vice versa.

For example, in constructing a shopping complex, the expected time of completion can be calculated using the time estimates of various activities. But if the construction has to be finished earlier, it requires additional cost to complete the project. We need to arrive at a time/cost trade-off between total cost of project and total time required to complete it.

Normal time:

Normal time is the time required to complete the activity at normal conditions and cost.

Crash time:

Crash time is the shortest possible activity time; crashing more than the normal time will increase the direct cost.

Cost Slope

Cost slope is the increase in cost per unit of time saved by crashing. Alinear cost curve is shown in Figure 8.27.



Example

An activity takes 4 days to complete at a normal cost of Rs. 500.00. If it is possible to complete the activity in 2 days with an additional cost of Rs. 700.00, what is the incremental cost of the activity?

Incremental Cost or Cost Slope
$$=\frac{C_c - N_c}{N_t - C_t} = \frac{700 - 500}{4 - 2} = Rs. 100.00$$

It means, if one day is reduced we have to spend Rs. 100/- extra per day.

Project Crashing

Procedure for crashing

Step1: Draw the network diagram and mark the Normal time and Crash time.

Step2: Calculate TE and TL for all the activities.

Step3: Find the critical path and other paths.

Step 4: Find the slope for all activities and rank them in ascending order.

Step 5: Establish a tabular column with required field.

Step 6: Select the lowest ranked activity; check whether it is a critical activity. If so, crash the activity, else go to the next highest ranked activity. Note: The critical path must remain critical while crashing.
Step 7: Calculate the total cost of project for each crashing
Step 8: Repeat Step 6 until all the activities in the critical path are fully crashed.

Example

The following Table 8.13 gives the activities of a construction project and other date

Activity	Nori	nal	Crash			
	Time (days)	Cost (Rs)	Time (days)	Cost (Rs)		
1-2	6	50	4	80		
1-3	5	80	3	150		
2-4	5	60	2	90		
2-5	8	100	6	300		
3-4	5	140	2	200		
4-5	2	60	1	80		

If the indirect cost is Rs. 20 per day, crash the activities to find the minimum duration of the project and the project cost associated.

Solution

From the data provided in the table, draw the network diagram (Figure 8.28) and find the critical path.



From the diagram, we observe that the critical path is 1-2-5 with project duration of 14days

The cost slope for all activities and their rank is calculated as shown in Table 8.14

Crash cost C, – Normal cost N,	Activity	Cost Slope	Rank
$Cost slope = \frac{C}{Normal time N} - Crash time C$	1-2	15	2
$t_t = 0$	1-3	35	4
	2-4	10	1
Cost Slope for activity $1 = 2 = \frac{80 - 50}{2} = \frac{30}{2} = 15$	2-5	100	5
6-4 2 $6-4$ 2	3-4	20	3
	4-5	20	3

Table 8.14: Cost Slope and Rank Calculated

The available paths of the network are listed down in Table 8.15 indicating the sequence of crashing (see Figure 8.29).

Path	Number of days crashed
1-2-5	14 -12 -11 10
1-2-4-5	13 – 11 10
1-3-4-5	12 - 11 - 10

Table 8.15: Sequence of Crashing



The sequence of crashing and the total cost involved is given in Table 8.16 Initial direct cost = sum of all normal costs given = Rs. 490.00

Figure 8.29: Network Diagram Indicating Sequence of Crashing

Activity Crashed	Project Duration	Critical Path	Direct Cost in (Rs.)	Indirect Cost in (Rs.)	Total Cost in (Rs.)
-	14	1-2-5	490	14 x 20 = 280	770
1 - 2(2) 2 - 5(2) 2 - 4(1) 3 - 4(2)	10	1 – 2 – 5 1 – 3 – 4 – 5 1 – 2 – 4 – 5	490 + (2 x 15) + (2 x 100) + (1 x 10) + (2 x 20) = 770	10 x 20 = 200	970

It is not possible to crash more than 10 days, as all the activities in the critical path are fully crashed. Hence the minimum project duration is 10 days with the total cost of Rs. 970.00.

Assignment

 You are required to prepare a network diagram for constructing a 5 floor apartmer The major activities of the project are given as follows:

Activity	Description	Immediate Predecessor
А	Selection of site	- 1 26
В	Preparation of drawings	4
С	Arranging the for finance	А
D	Selection of contractor	A
E	Getting approval from Govt	А
F	Laying the foundation	E
G	Start construction	D, F
Н	Advertise in newspaper	B, C
I	Allocation of tenants	G, H

 For the problem No.1 the time estimates in days are given. Determine the Time earliest and Time latest, and the critical activities

Activity	Α	В	С	D	E	F	G	н	Ι
Time (days)	3	5	7	2	5	20	60	2	10

Draw a network diagram for the project:

Activity	Α	В	С	D	Ε	F	G	Н	Ι	J
Predecessor	-	А	В	В	В	С	С	F, G	D, E, F	Ι

A national conference is planned in a college. The activities are listed down along with their predecessors and time taken. Prepare a network diagram and determine the critical activities.

Activity	Description	Immediate	Duration (days)
		Predecessor	
A	Confirm lead speaker and topic	-	5
В	Prepare brochure	-	1
С	Send letters to other speakers	В	2
D	Get confirmation from speakers	С	5
E	Send letters to participants	C,D	2
F	Obtain travel plans from speakers	D	2
G	Arrange for accommodation for speakers	F	1
Н	Get handouts from speakers	F	4
I	Finalize registrations	G,H	10
J	Arrange hall and AV	I	1
К	Conduct of programme	J	1

For the PERT problem find the critical path and project duration. What is the probability that the project will be completed in 25 days?

Activity	Predecessor	Time		
		Optimistic	Mostlikely	Pessimistic
А	-	2	5	14
В	-	1	10	12
С	А	0	0	6
D	А	1	4	7
E	С	3	10	15
F	D	3	5	7
G	В	1	2	3
Н	E,F	5	10	15
I	G	3	6	9

The following	table lists the	jobs of a network	along with their estimates.
		5	0

Activity	Time (Weeks)		Cast (Rs)	
	Normal	Crash	Normal	Crash
1-2	9	4	1300	2400
1-3	15	13	1000	1380
2-3	7	4	7000	1540
2-4	7	3	1200	1920
2-5	12	6	1700	2240
3-6	12	11	600	700
4-5	6	2	1000	1600
5-6	9	6	900	1200

a. Draw the project network diagram.

- b. Calculate the length and variance of the critical path.
- c. What is the probability that the jobs on the critical path can be completed in 41 days?

